

PHYS393: Important Equations

2011 - 2012

Those that need to be memorised are labelled 'Memorise'. The rest you should learn how to derive and explain.

There are about 90 equations here.

About 30 need to be memorised. Most of these you would already know, like the first one below.

The rest should hopefully become clear once you understand the meaning of each quantity in the equations.

Handout 1

Handout 1, slide 8 - Memorise

$$dQ = TdS$$

Handout 1, slide 11 - Memorise

$$E = \frac{\hbar^2 k^2}{2m}$$

Handout 1, slide 12 - Memorise

$$p_x = \hbar k$$

Handout 1, slide 16 - Memorise

$$N = n_1 + n_2 + \dots$$

$$E = n_1 \varepsilon_1 + n_2 \varepsilon_2 + \dots$$

Handout 1, slide 20 - Memorise

$$n_i = A \exp(-\varepsilon_i/k_B T)$$

Handout 1, slide 23 - Memorise

$$\varepsilon = -\mu_B B \text{ or } +\mu_B B$$

Handout 1, slide 25 - Memorise

$$C = \frac{dU}{dT}$$

Handout 1, slide 27 - Memorise

$$Z_{SP} = \sum \exp(-\varepsilon_i/k_B T)$$

Handout 1, slide 29 - Memorise

$$p_i = \frac{n_i}{N} = \frac{\exp(-\varepsilon_i/k_B T)}{Z}$$

Handout 1, slide 31

$$S = \int \frac{dU}{T} = \int \frac{C dT}{T}$$

Handout 1, slide 32 - Memorise

$$S = k_B \ln \Omega$$

Handout 1, slide 35 - Memorise

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

$$C = \frac{dU}{dT} = \frac{3}{2}Nk_B$$

Handout 1, slide 36 - Memorise

$$k_x = \frac{n_x \pi}{a}$$

Handout 1, slide 39 - Memorise

$$U = \int n(\varepsilon) \varepsilon g(\varepsilon) d\varepsilon.$$

Handout 1, slide 42

$$G(k) = \frac{V k^3}{6\pi^2}$$

$$g(k) = \frac{dG(k)}{dk} = \frac{V k^2}{2\pi^2}$$

Handout 1, slide 42 - Memorise

$$g(k) = \frac{dG(k)}{dk}$$

Handout 1, slide 43 - Memorise

$$g_\varepsilon(\varepsilon)d\varepsilon = g_k(k)dk$$

Handout 1, slide 44

$$g(\varepsilon) = \frac{4m\pi V}{h^3}(2m\varepsilon)^{1/2}$$

Handout 1, slide 45

$$G(\varepsilon) = \frac{4\pi V}{3h^3}(2m\varepsilon)^{3/2}.$$

Handout 1, slide 51 - Memorise

$$n(\varepsilon)d\varepsilon = Ag(\varepsilon) \exp(-\varepsilon/k_B T)d\varepsilon$$

$$N = A \int g(\varepsilon) \exp(-\varepsilon_i/k_B T)d\varepsilon$$

Handout 1, slide 52

$$Z_{SP} = \int g(\varepsilon) \exp(-\varepsilon/k_B T) d\varepsilon$$

Handout 1, slide 52

$$Z_{SP} = V \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2}$$

$$A = \frac{N}{V} \left(\frac{h^2}{2\pi m k_B T} \right)^{3/2}$$

Handout 1, slide 53

$$n(v)dv = n(\varepsilon)d\varepsilon$$

Handout 1, slide 57 - Memorise

$$k_B = \frac{R}{N_A}$$

Handout 2

Handout 2, slide 10 - Memorise

$$f(\varepsilon) = \frac{n(\varepsilon)}{g(\varepsilon)} = \frac{1}{\exp((- \mu + \varepsilon)/k_B T) + 1}$$

Handout 2, slide 12

$$N = 2 \times \int_0^\infty n(\varepsilon) d\varepsilon$$

Handout 2, slide 13 - Memorise

$$n(\varepsilon) = g(\varepsilon)f(\varepsilon)$$

$$N = 2 \times \int_0^{E_F} g(\varepsilon) d\varepsilon$$

Handout 2, slide 14

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3}$$

Handout 2, slide 30

$$U_1 = \frac{3}{2} N_1 k_B T$$

Handout 2, slide 31

$$d\varepsilon = k_B T$$

Handout 2, slide 32

$$N_1 \approx 2g(E_F)k_B T$$

Handout 2, slide 36

$$C = \frac{\pi^2}{2} N k_B \frac{T}{T_F}$$

Handout 2, slide 37

$$C = \gamma T$$

$$c_V = \gamma T + AT^3$$

Handout 2, slide 48 - Memorise

$$T_F = \frac{E_F}{k_B}$$

Handout 3

Handout 3, slide 10 - Memorise

$$n(\varepsilon)d\varepsilon = \frac{g(\varepsilon)d\varepsilon}{\exp(\varepsilon/k_B T) - 1}$$

Handout 3, slide 12 - Memorise

$$\varepsilon = \hbar\omega$$

$$v = \frac{\omega}{k}$$

Handout 3, slide 13 - Memorise

$$g(\omega)d\omega = g(k)dk$$

Handout 3, slide 13

$$g(\omega) = \frac{V\omega^2}{2\pi^2v^3}$$

Handout 3, slide 14

$$g(\omega)d\omega = 3 \times \frac{V\omega^2d\omega}{2\pi^2v^3}$$

Handout 3, slide 15

$$\int_0^{\omega_D} g(\omega)d\omega = 3N$$

Handout 3, slide 16

$$\omega_D = \left(\frac{6N\pi^2v^3}{V} \right)^{1/3}$$

Handout 3, slide 18

$$U = \int_0^{\omega_D} \hbar \omega n(\omega) d\omega = \int_0^{\omega_D} \frac{\hbar \omega g(\omega) d\omega}{\exp(\hbar \omega / k_B T) - 1}$$

Handout 3, slide 28

$$g(\omega) d\omega = 2 \times \frac{V \omega^2 d\omega}{2\pi^2 c^3}$$

Handout 3, slide 29

$$n(\omega) d\omega = 2 \times \frac{V \omega^2 d\omega}{2\pi^2 c^3} \times \frac{d\omega}{\exp(\hbar \omega / k_B T) - 1}$$

Handout 4

Handout 4, slide 47

$$H_D - H_C = T(S_D - S_C) = T(106T - 22T) = 84T^2 \text{ J/mol}$$

Handout 4, slide 48

$$\dot{Q} = \dot{n}_3 \Delta H$$

Handout 4, slide 48

$$\dot{Q} = 84 \dot{n}_3 T^2 \text{ W}$$

Handout 4, slide 71 - Memorise

$$\frac{B}{T} = \text{constant}$$

Handout 4, slide 80

$$T_f = \frac{T_i}{B_i} \sqrt{B_f^2 + b^2}$$

Handout 4, slide 82

$$\varepsilon_d = \mu b.$$

$$k_B T_c = \mu b$$

Handout 5

Handout 5, slide 6 - Memorise

$$E = p^2/2m$$

Handout 5, slide 8 - Memorise

$$E = pc$$

Handout 5, slide 15 - Memorise

$$v_L = \left(\frac{E}{p} \right)_{min}$$

Handout 6

Handout 6, slide 9 - Memorise

$$N = \int_0^\infty \frac{g(\varepsilon)d\varepsilon}{\exp((\varepsilon - \mu)/k_B T) - 1}$$

Handout 6, slide 10 - Memorise

$$f(\varepsilon) = \frac{1}{\exp((\varepsilon - \mu)/k_B T) - 1}$$

Handout 6, slide 15

$$N_{ex} = \int_0^\infty \frac{g(\varepsilon)d\varepsilon}{\exp(\varepsilon/k_B T) - 1}$$

Handout 6, slide 15

$$N_{ex} = \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2} 2.612V$$

Handout 6, slide 16

$$T_{BE} = \frac{h^2}{2\pi m k_B} \left(\frac{N}{2.612V} \right)^{2/3}$$

Handout 6, slide 19

$$U = \int_0^\infty \frac{\varepsilon g(\varepsilon) d\varepsilon}{\exp(\varepsilon/k_B T) - 1}$$

Handout 6, slide 19

$$U = 0.7704 k_B N \frac{T^{5/2}}{T_{BE}^{3/2}}$$

Handout 6, slide 20

$$C = 1.926 k_B N \left(\frac{T}{T_{BE}} \right)^{3/2}$$

Handout 6, slide 45

$$v_L = \frac{\Delta}{p_0}$$

Handout 6, slide 46

$$v = \frac{\hbar k}{m}$$

Handout 6, slide 48

$$v = \frac{\hbar}{m} \frac{d\phi}{dx}$$

Handout 6, slide 50

$$\int \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} 2n\pi = n \frac{h}{m}$$

Handout 6, slide 58

$$f(0) = \frac{1}{\exp(-\mu/k_B T) - 1}$$

Handout 6, slide 59

$$\int_0^\infty \frac{g(\varepsilon) d\varepsilon}{\exp(\varepsilon/k_B T) - 1} = \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2} 2.612V$$

Handout 7

Handout 7, slide 21

$$\hbar \Delta \phi = \frac{m}{\rho q} \int_L \mathbf{J} \cdot d\mathbf{l} + q\Phi$$

$$\frac{m}{\rho q} \int_L \mathbf{J} \cdot d\mathbf{l} = -q\Phi$$

Handout 7, slide 29

$$J = \frac{\text{current}}{\text{area}} = \frac{NI}{L\lambda_s}$$

Handout 7, slide 30

$$\lambda^2 = \frac{m}{\mu_0 q^2 \rho}$$

Handout 7, slide 39

$$\Phi = \frac{h}{2e}$$

Handout 7, slide 42

$$\Phi = \frac{nh}{e},$$

Handout 7, slide 52

$$C_v = D \exp\left(-\frac{\Delta}{k_B T}\right)$$

Handout 7, slide 53

$$\frac{2\Delta}{k_B T_c} = 3.52$$

Handout 7, slide 54 - Memorise

$$k_B T_c \approx \Delta$$

Handout 8

Handout 8, slide 27

$$2\Delta = 3.52 k_B T_c$$